THICKNESS OF THE LAYER OF LUBRICANT AND THE RESISTANCE TO ROLLING OF CYLINDERS IN ELASTOHYDRODYNAMIC CONTACT

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Equations for calculating the mean thickness of a layer of lubricant of cylinders in elastohydrodynamic contact are available which have been obtained by three different methods: 1) by the method described in [1]; 2) by numerical solution of the equations of the elastohydrodynamic theory of lubrication in the entrance and exit regions and by further matching of the results with the distributed pressure in the middle part of the contact [2]; and 3) by processing experimental results. A review of the majority of the existing equations and new equations of the third group is given in [3]. The problem of the resistance to rolling of two elastic cylinders separated by a layer of viscous liquid has been investigated only slightly. In this paper we present equations for the average and minimum thickness of the layer of lubricant, and also for the resistance to rolling obtained by processing the results of a direct numerical solution of the equations of the isothermal elastohydrodynamic problem.

1. The equations of the one-dimensional isothermal problem of the elastohydrodynamic theory of lubrication have the form [4]

$$\frac{d}{dx}\left[h^3 \exp(-Qp)\frac{dp}{dx}\right] = \frac{V}{H_0^2}\frac{dh}{dx}; \tag{1.1}$$

$$H_0(h-1) = x^2 - c^2 + \frac{2}{\pi}\int_a^c p(t)\ln\frac{c-t}{|t-x|}\,dt. \tag{1.2}$$

The boundary conditions are

$$p(a) = p(c) = (dp/dx)(c) = 0. \tag{1.3}$$

In addition

$$\int_a^c p(t)\,dt = \pi/2. \tag{1.4}$$

In (1.1)–(1.4) x, t, a, and c refer to the half-width of the Hertz contact $b = [8qR/(\pi E')]^{1/2}$, q is the load per unit length of the cylinder, $E' = E/(1-\nu^2)$, E is the elastic modulus, ν is Poisson's ratio, and $1/R = 1/R_1 + 1/R_2$, where R_1 and R_2 are the radii of the cylinders. The unknown contact pressure $p(x) \geq 0$ is referred to the maximum Hertz pressure $p_0 = [qE'/2\pi R)]^{1/2}$, the profile of the gap $h(x) > 0$ is referred to the thickness of the layer of lubricant $h_0 = h(c)$ at the exit from the contact region, $H_0 = 2h_0R/b^2$; $V = 3\pi^2[\mu_0 \cdot (u_1 + u_2)/(2E'R)](E'R/q)^2$, u_1 and u_2 are the velocities of the cylinder surfaces, μ_0 is the viscosity of the lubricant at room pressure, $Q = \alpha p_0$, α is the piezocoefficient in the equation for the viscosity $\mu = \mu_0 \exp(\alpha\bar{p})$, $\bar{p}$ is the physical pressure, and a and c are the coordinates of the beginning and end of the region of positive pressure. The parameters M_0 and c are unknown and must be determined during the solution from the additional conditions (1.3) and (1.4).

In [4] a numerical method is described for solving Eqs. (1.1)–(1.3) in which the spline approximation of the pressure p(x) is employed. Equations (1.1)–(1.3) in this case reduce to a system of nonlinear equations which are solved by Newton's method. Calculations [4] show that the method is efficient.

When solving Eqs. (1.1)–(1.3) over a wide range of variation of the parameters V, Q, and a difficulties arise due to the large amount of computer time required. For a number of nodes of the difference net N = 90 version on the BÉSM-6 computer takes about two hours.

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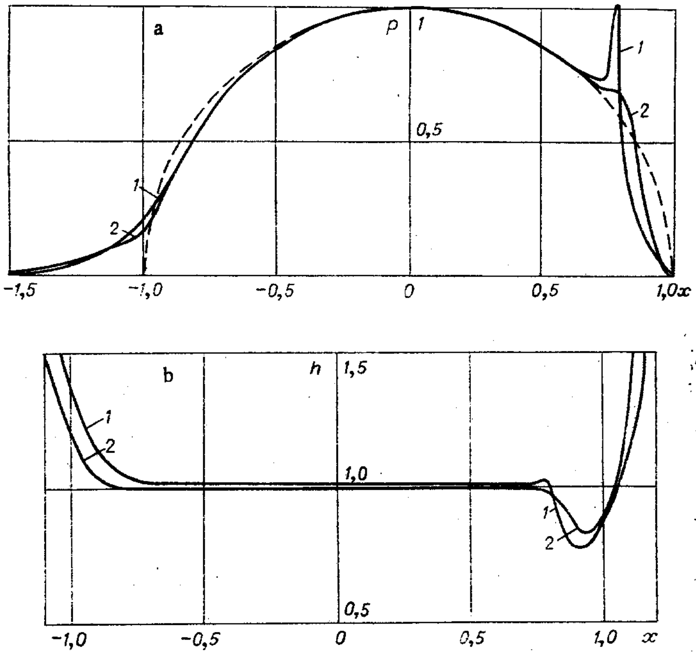

Fig. 1

When $Q \gg 1$ Eq. (1.1) can be linearized [5]

$$\frac{d}{dx}\left[h_0^3(x)\exp\left(-Qp_0\sqrt{1-x^2}\right)\frac{dp}{dx}\right]=\frac{V}{H_0^2}\frac{dh}{dx}, \tag{1.5}$$

where $h_0(x) = 1 + H_0^{-1}[|x|\sqrt{x^2-1} - \ln(|x| + \sqrt{x^2-1})]\,\theta(x^2-1)$ (θ is the Heaviside function). The application of the method described in [4] to (1.5), (1.2), and (1.3) leads to a system of linear equations. A comparison of the solutions of (1.1)-(1.4) and (1.5), (1.2)-(1.4) for $Q = 7.5$ and $V = 0.11$ is given in Fig. 1 (*a* is the pressure distribution and b is the profile of the gap). Curves 1 and 2 are the solutions of the nonlinear and linear problems. The dashed curve in Fig. 1a corresponds to the Hertz pressure distribution. The table compares H_0 and $S = (2/\pi)\int_a^c p(x)\,x\,dx$, obtained by solving the nonlinear system (1.1)-(1.4) (subscript 1) and the linearized system (1.5), and (1.2)-(1.4) (the subscript 2); H_{00} is calculated by the method described in [1].

It can be seen that to determine the thickness of the layer of lubricant one can obtain good accuracy by using the solution of the linearized system. The error in the value of S connected with the moment of the resistance to rolling may reach 15%. Linearization of Eq. (1.1) preserves the main features of the nonlinear problem. A second pressure maximum also appears in the solution of the linear problem for $Q \geq 10$ [4]. Calculation using version (1.5), and (1.2)-(1.4) for $N = 90$ takes 10-15 min on the BÉSM-6 computer.

All later results in this paper are based on the numerical solution of the linearized system (1.5) and (1.2)-(1.4).

2. The average thickness of the layer of lubricant, which is of considerable interest in practice, differs only slightly from h when $x = c$ (see Fig. 1b).

In Fig. 2 the triangles represent some results of the solution of (1.5) and (1.2)-(1.4), the continuous lines were obtained by approximating these results, line 1 corresponds to $Q = 5$, line 2 corresponds to $Q = 10$, line 3 corresponds to $Q = 20$, and the dashed line corresponds to the approximate result obtained in [1]: $H_0 = 0.254\,(VQ)^{0.727}$. The dependence of H_0 on V and Q has a pronounced power form. The mean-square approximation of 11 points (V, Q, and H_0) in the range $5 \leq Q \leq 20$, $0.005 \leq V \leq 0.2$, with $a = -2$ is obtained from the equation

$$H_0 = 0.53V^{0.6}Q^{0.3}. \tag{2.1}$$

The maximum error of the approximation is 5%. In dimensional variables Eq. (2.1) has the form

$$h_0/R = 3.57(\mu_0 u/E'R)^{0.6}(\alpha E')^{0.3}(p_0/E')^{-0.1},$$

where $u = (u_1 + u_2)/2$. For comparison we present the result obtained in [3] by processing experimental data

TABLE 1

Q	V	H_{01}	H_{02}	H_{00}	S_1	S_2
3,9	0,17	0,261	0,276	0,188	0,05	0,052
7,5	0,11	0,252	0,258	0,221	0,034	0,029
7,24	0,076	0,203	0,200	0,165	0,032	0,028

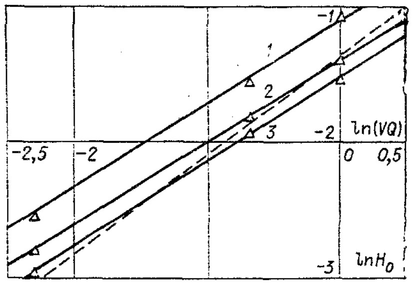

Fig. 2

$$h_0/R = 5.8(\mu_0 u/E'R)^{0.7}(\alpha E')^{0.5}(p_0/E')^{-0.2}.$$

The equation [6]

$$h_0/R = 4.05(\mu_0 u/E'R)^{0.75}(\alpha E')^{0.6}(p_0/E')^{-0.2} \tag{2.2}$$

is based on replacing the elastic bodies by a Winkler medium and is obtained by a special choice of the Winkler constant.

In addition to the average thickness of the layer of lubricant it is also important to determine the minimum thickness. On the basis of the results of a solution of (1.5) and (1.2)–(1.4) in the above range of V and Q we obtain the following equation for $H_{min} = H_0 h_{min}$:

$$H_{min} = 0.31 V^{0.66} Q^{0.52}. \tag{2.3}$$

In dimensional variables (2.3) has the form

$$h_{min}/R = 2.05(\mu_0 u/E'R)^{0.66}(\alpha E')^{0.52}(p_0/E')^{-0.12}.$$

The following result is obtained in [2]:

$$h_{min}/R = 1.26(\mu_0 u/E'R)^{0.7}(\alpha E')^{0.6}(p_0/E')^{-0.26},$$

which contradicts Eqs. (1.1)–(1.4) since it does not satisfy the condition $H_{min} = H_{min}(V, Q)$.

3. We will consider the problem of determining the force and moment of the resistance to rolling which acts on unit length of the cylinder in the case of pure rolling and when the cylinders are made of the same materials. The thickness of the layer of lubricant in dimensional variables will be represented by the equations

$$h = h_1 + h_2, \quad h_1 = \frac{h^*}{2} + \frac{x^2}{2R_1} - \frac{2}{\pi E'}\int_a^c p(t)\ln|t-x|\,dt,$$
$$h_2 = \frac{h^*}{2} + \frac{x^2}{2R_2} - \frac{2}{\pi E'}\int_a^c p(t)\ln|t-x|\,dt, \tag{3.1}$$

where h^* is a certain constant (Fig. 3).

A supporting force $F_y = p dx$, a drag resistance force $F_x = p(dh_1/dx)$, and a viscous friction force $F_\mu = \frac{h_1+h_2}{2}\frac{dp}{dx}\,dx$ act on an element dx of the surface of the upper cylinder. The principal vector F_1 and the principal moment M_1 with respect to the instantaneous center of rolling are

$$F_1 = \int_a^c p\frac{dh_1}{dx}\,dx + \int_a^c \frac{h_1+h_2}{2}\frac{dp}{dx}\,dx; \tag{3.2}$$

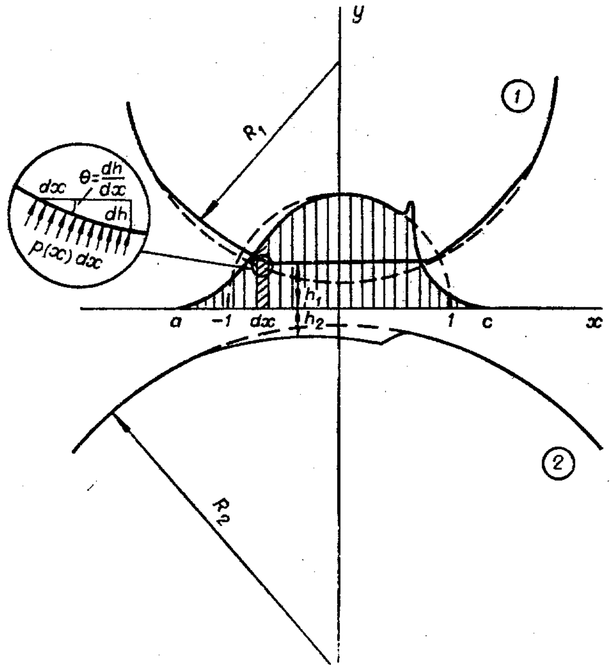

Fig. 3

$$M_1 = -\int_a^c pxdx. \tag{3.3}$$

Substituting (3.1) into (3.2) and integrating by parts we obtain

$$F_1 = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\int_a^c pxdx. \tag{3.4}$$

Equations (3.2)-(3.4) are written in dimensional variables. If we introduce the dimensionless force $F_1' = F_1R/(qb)$, the dimensionless moment $M_1' = M_1/(qb)$, and omit the primes, we obtain

$$F_1 = \frac{R_2 - R_1}{\pi(R_1 + R_2)}\int_a^c pxdx, \quad M_1 = \frac{2}{\pi}\int_a^c pxdx.$$

As in the case of (2.1) and (2.2) we can approximate the dependence of M_1 on V and Q with an error of not more than 6% by the equation

$$M_1 = 0{,}28(V/Q)^{0.54}. \tag{3.5}$$

We obtain for F_1

$$F_1 = 0.14\frac{R_2 - R_1}{R_1 + R_2}\left(\frac{V}{Q}\right)^{0.54}. \tag{3.6}$$

For the lower cylinder

$$F_2 = -F_1, \; M_2 = M_1. \tag{3.7}$$

In dimensional variables Eqs. (3.5) and (3.6) have the form

$$\frac{F_1}{E'R} = 3.01\,\frac{R_2 - R_1}{R_1 + R_2}\left(\frac{\mu_0 u}{E'R}\right)^{0.54}(\alpha E')^{-0.54}\left(\frac{p_0}{E'}\right)^{0.3}; \tag{3.8}$$

$$M_1/E'R^2 = 6.02(\mu_0 u/E'R)^{0.54}(\alpha E')^{-0.54}(p_0/E')^{0.3}. \tag{3.9}$$

In [7] results are presented of the approximation of the solution obtained in [3], from which we find

$$\frac{F_1}{E'R} = 9.2\,\frac{R_2 - R_1}{R_1 + R_2}\left(\frac{\mu_0 u}{E'R}\right)^{0.7}(\alpha E')^{-0.3}; \tag{3.10}$$

$$M_1/E'R^2 = 18.4(\mu_0 u/E'R)^{0.7}(\alpha E')^{-0.3}. \tag{3.11}$$

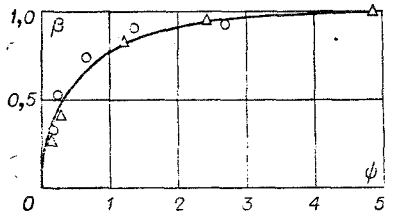

Fig. 4

A drawback of Eqs. (3.10) and (3.11) is the fact that the effect of the load on the resistance to rolling is ignored. This effect has been detected experimentally [8] and also by a more correct solution of the initial equations (1.1)-(1.4).

A comparison of Eqs. (3.8) and (3.9) with experimental data [8] shows that the calculation corresponds to experiment up to rolling speeds of the order of 5 m/sec. The index in the relation between the force of friction and the rolling speed, calculated from the data given in [8], is ≈ 0.4, and the index for the contact pressure ≈ 0.18–0.28.

In experiments on a disk system [8] so-called friction rolling was set up when one disk (on which the moment of the rolling resistance was measured) is driving while the driven disk is set so that the resistance to rotation in its supports is practically zero. It follows from (3.7) and (3.8) that for $R_1 = R_2$, $F_1 = F_2 = 0$, while $M_1 = M_2 = M$. For equilibrium (stationary rotation) of the driving disk some slipping of the disks and a friction force due to this slipping are necessary. The moment of this force with respect to the axis of the driven disk must be M. On the driving disk the force of friction is directed in the opposite direction and causes a twofold increase in the moment of the resistance to rolling with respect to the axis of the disk. The moment calculated taking these factors into account for similar disks of radius R = 70 mm and width $l = 8$ mm for a rolling speed u = 1 m/sec, a contact pressure $p_0 = 0.45$ GN/m^2, and a temperature T = 30°C under conditions of abundant lubrication with MS-20 oil is 0.113 N·m. The measured moment is 0.187 N·m. The considerable disagreement between theory and experiment for u > 5 m/sec is obviously due to nonisothermal flow of the lubricant.

The coordinate a of the point where the region of positive pressure begins has a small effect on the results of the calculation if $a \ll -1$. On the other hand, if $a \gg -1$, the solution will depend on a. By setting different values of a we can simulate the rolling under conditions when there is insufficient lubricant in the contact zone (so-called low-grade lubricant or oil starvation [9]). Figure 4 shows the results of a solution of Eqs. (1.5) and (1.2)-(1.4) for several values of β. Along the abscissa axis we have plotted the parameter [9] $\psi = |a + 1|\, H_\infty^{-2/3}$, where $H_\infty = \lim_{a\to-\infty} H_0$, and along the ordinate axis we have the plotted the ratio $\beta = H_0/H_\infty$. The small circles correspond to Q = 5 and V = 0.1, and the triangles correspond to Q = 10 and V = 0.1. The continuous curve is obtained by the method described in [1] in [9]. The value of a can be found experimentally from the position of the boundary of the region occupied by the lubricant. As $a \to -1$ the pressure distribution becomes close to the Hertz solution for elastic contact [4].

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SOME MODEL CALCULATIONS OF FRICTIONAL RESISTANCE IN THE MOTION OF BODIES WITH BOUNDARY LAYERS OF VARIABLE VISCOSITY

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Flows when a variable viscosity is present in the boundary layer are of great interest from the applied aspect. In the opinion of a number of authors [1, 2], the motions of marine animals, for which mucus emerges as a substance reducing the viscosity of aqueous solutions, can also serve as analogs of such motions. Some investigations devoted to these questions have been published in [3].

In the present report we give the results of a theoretical investigation of the possible decrease in frictional resistance during flow of the Couette type and during steady and nonsteady flow over a flat plate when at its surface one assigns a concentration of some substance capable of reducing the viscosity of the solution which forms.

1. Let the viscosity vary by the law (Fig. 1)

$$\nu/\nu_0 = \begin{cases} 1 & \text{for } 0 \leqslant |y| \leqslant 1-\alpha, \\ \mathrm{ch}^{-1}\frac{k}{\alpha}(y-1+\alpha) & \text{for } 1-\alpha \leqslant |y| \leqslant 1, \end{cases}$$

where α is the thickness of the diffusional boundary layer; k is some number for which the relative viscosity near the surface is minimal and equal to $\nu/\nu_0|_{y=1} = 1/\cosh k$.

First of all, let us consider flow of the Couette type. In this case

$$\frac{d}{du}\left(\nu\frac{du}{dy}\right) = 0, \quad u(1) = 1, \quad \frac{du(0)}{dy} = 0.$$

The solution has the form

$$u = \frac{1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,\frac{k}{\alpha}(y-1+\alpha)}{1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,k}.$$

Hence,

$$\tau/\tau_0 = \left(1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,k\right)^{-1},$$

where τ_0 is the frictional stress when $\nu = \nu_0$. One can ascertain that $\tau/\tau_0 < 1$ when α, k > 0. However, when $\alpha = 0.1$ and k = 1.7, which corresponds to $\nu/\nu_0|_{y=1} = 0.35$, the relative friction is $\tau/\tau_0 = 0.95$. When $\alpha = 0.1$ and k = 3 ($\nu/\nu_0|_{y=1} = 0.1$), $\tau/\tau_0 = 0.83$; when $\alpha = 0.1$ and k = 3.7 ($\nu/\nu_0|_{y=1} = 0.05$), $\tau/\tau_0 = 0.69$.

Thus, an approximately threefold decrease in the viscosity near the surface is necessary for a significant decrease in resistance which could be noticed experimentally (by 5%).

A similar result is obtained in an analysis of Poiseuille flow.

2. Let us consider the case of nonsteady motion. Imagine an infinite plate suddenly set into motion.

The equation of motion and the equation for the diffusion in the boundary layer have the form

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